A Customizable Reflectance Model for Everyday Rendering

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Abstract : This paper introduces a new reflectance model intended for realistic rendering, that includes three main features. First, it is fast and simple though it obeys to the main laws of physics (Energy conservation law, Helmholtz reciprocity rule, Microfacet theory, Fresnel equation). Second, it is defined by a small number of parameters which can be specified either intuitively or related to experimental measurements. Third, it is expressed by a formulation of varying complexity that can be customized according to the number of physical phenomena the user wants to include (isotropic or anisotropic reflection, homogeneous or heterogeneous materials, spectral modifications, surface self-shadowing).

Keywords : Bidirectional Reflectance Distribution Function, Isotropic and Anisotropic Reflectance Model

1 Introduction

Reflectance models currently used in computer graphics can be divided in two main families : either *empirical models* [PHON75] [BLIN77] which are computationally inexpensive but are lacking of physical validity, or *theoretical models* [COOK81] [KAJI85] [HE91] which are expensive and usually unnecessarily accurate compared to the error generated by other stages of the rendering pipeline (global illumination, sampling, interpolation). Such a contradictory situation has been noticed by Ward who proposed a kind of intermediary model primarily intended to fit experimental data [WARD92].

This paper goes a step further in the same direction by proposing a reflectance model that can be customized according to the number of physical phenomena the user wants to include. In Section 2, several definitions and notations are provided. Section 3 recalls the formulation of some previous reflectance models while Section 4 discusses about unsatisfactory points existing in these models. Finally, the new model is detailled and experimented in Section 5 and 6.

2 Bidirectional Reflectance Distribution Function

The interaction of light with a surface is usually expressed using a function called *bidirectional reflectance distribution function* (BRDF for short) that relates an incoming and an outcoming radiance at a given point on the surface (see Figure 1) :

$$L_\lambda(P,V) = \int_{V' \in \mathcal{V}} R_\lambda(P,V,V')\; L_\lambda(P,-V')\; (N \cdot V')\; dV' \tag{1}$$

where

- $L_\lambda(P,V)$ is the reflected radiance leaving point P in direction V
- $L_\lambda(P,-V')$ is the incident radiance reaching point P from direction $-V'$
- $R_\lambda(P,V,V')$ is the BRDF of the surface at point P between directions V and V'
- dV' is a differential solid angle surrounding direction V'
- $\mathcal{V}$ is the set of possible directions for the incident light (*ie* the hemisphere above the surface)

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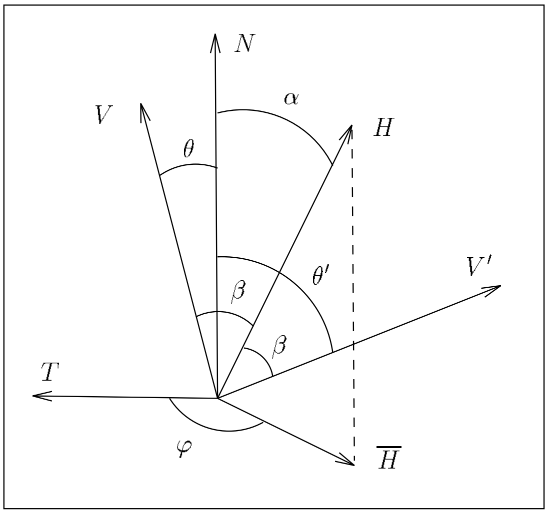
Equation 1 is a monochromatic equation expressed for a given wavelength λ. In the present paper, we use the following notation convention : every term that is function of the wavelength will be subscripted by λ. Such a term has to be defined and/or computed, theoretically for every wavelength of the visible spectrum, and practically for a given number of samples (at least three in the RGB model).

The BRDF has got two important properties that result directly from physics of light [BECK63]. First, due to the *Helmholtz Reciprocity Rule*, R_λ is symmetric relative to V and V' :

$$\forall\, V \in \mathcal{V} \quad \forall\, V' \in \mathcal{V} \quad R_\lambda(P,V,V') = R_\lambda(P,V',V) \tag{2}$$

Second, due to the *Energy Conservation Law*, R_λ has to fulfill the normalization condition :

$$\forall\, V \in \mathcal{V} \quad \int_{V'\in\mathcal{V}} R_\lambda(P,V,V')\ (N\cdot V')\ dV' \leq 1 \tag{3}$$



N : Surface normal vector

T : Surface tangent vector

V : Outcoming direction

V' : Incoming direction

H : Facet normal vector

$\overline{H}$: Projection of $H \perp N$

Figure 1 : Angles and vectors for BDRF definition

The different formulations of the BRDF presented in the next sections will be expressed using the following notations :

α	=	$< H, N >$	t	=	$\cos\alpha$
β	=	$< V, H >$	u	=	$\cos\beta$
θ	=	$< V, N >$	v	=	$\cos\theta$
θ'	=	$< V', N >$	v'	=	$\cos\theta'$
φ	=	$< \overline{H}, N >$	w	=	$\cos\varphi$

3 Previous Work

3.1 Isotropic BRDF

When the BRDF at a point P does not change while the surface is rotated around its normal vector at P (*ie* the BRDF does not depend on angle φ), the surface is called *isotropic*. According to the shape of the BRDF, two kinds of surfaces are traditionally distinguished :

Diffuse surfaces : The light is reflected in every direction. The limit case — *perfectly diffuse surfaces* or *lambertian surfaces* — is obtained when the BRDF becomes a constant function (*ie* the light is equally reflected in every direction).

Specular surfaces : The light is reflected only in a small area around the mirror direction. The limit case — *perfectly specular surfaces* or *smooth surfaces* — is obtained when the BRDF becomes a Dirac function (*ie* the light is reflected in a single direction).

In the computer graphics field, the first reflectance model suited to non-lambertian has been proposed by Phong [PHON75] and later slightly modified by Blinn [BLIN77]. In that model, the BRDF depends only on the cosine of angle α (see Figure 1) and is expressed as a linear combination of a diffuse part and a specular one :

$$R_\lambda(t) = d\,D_\lambda + s\,S_\lambda\,t^n \quad \text{with} \quad d + s = 1 \tag{4}$$

where

- d (resp. s) $\in [0,1]$ is the ratio of the surface behaving as a diffuse (resp. specular) reflector
- D_λ (resp. S_λ) $\in [0,1]$ is the ratio of light reflected by the diffuse (resp. specular) reflector
- $n \in [1, \infty[$ characterizes the brightness of the specular reflector

The first theoretical reflectance model has been introduced in the computer graphics field by Cook & Torrance [COOK81] using work previously done in physics by Beckmann & Spizzichino [BECK63] and Torrance & Sparrow [TORR67] about the reflection of electromagnetic waves on rough surfaces. In that model, a surface is supposed to be composed of so-called *microfacets* which are small planar surfaces. Only microfacets whose normal vector is in the direction H (see Figure 1) contribute to the reflection between V and V'. As in the Phong model, the BDRF is expressed as a linear combination of a diffuse part and a specular one, but it depends here on the cosine of four different angles :

$$R_\lambda(t,u,v,v') = \frac{d}{\pi}\,D_\lambda + \frac{s}{4\pi v v'}\,D(t)\;F_\lambda(u)\;G(v,v') \quad \text{with} \quad d + s = 1 \tag{5}$$

where

- $d \in [0,1]$, $s \in [0,1]$ and $D_\lambda \in [0,1]$ have the same meaning as in Equation 4
- $D(t) \in [0, \infty[$ is the microfacets slope distribution function which defines the fraction of the facets that are oriented in the direction H
- $F_\lambda(u) \in [0,1]$ is the Fresnel factor which describes how light is reflected by each microfacet
- $G(v,v') \in [0,1]$ is the geometrical attenuation coefficient which expressed the ratio of light that is not self-obstructed by the surface

Several formulations have been proposed and compared to experimental results both for $D(t)$ and $G(v,v')$. When a gaussian behaviour is assumed for rough surfaces, $D(t)$ is given by Equation 6 [BECK63] and $G(v,v') = G(v)G(v')$ by Equation 7 [SMIT67] where m is the RMS slope of the microfacets :

$$D(t) = \frac{1}{m^2 t^4}\; e^{\frac{t^2-1}{m^2 t^2}} \tag{6}$$

$$G(v) = \frac{g}{g+1} \quad \text{with} \quad g = \sqrt{h\pi}\,(2 - \operatorname{erfc}\sqrt{h}) \quad \text{and} \quad h = \frac{v^2}{2m^2\,(1-v^2)} \tag{7}$$

Another theoretical reflectance model that accounts for even more physical phenomena (polarization, diffraction, interference) has been proposed by He *et al.* [HE91]. The model has quite a similar expression as the Cook-Torrance model ; the main differences are the addition of a coherent reflection term and a more complete (and much more complex) formulation of the distribution function.

3.2 Anisotropic BRDF

A surface is called *anisotropic* when the BRDF is function of the orientation of the surface along its normal (*ie* the BRDF depends on angle φ). Relatively few reflectance model accounting for anisotropy have been proposed in the computer graphics field. Two early brute force methods have been presented by Kajiya and Cabral *et al.* The first using a general Kirchhoff solution for scattering of electromagnetic waves [KAJI85] and the second using tabulated height fields to represent surface roughness at a microscopic level [CABR87]. Poulin & Fournier have proposed a model with a more reasonable cost, in which anisotropic facet orientations are simulated by adding or subtracting groups of microscopic cylinders on the surface.

A simple model for anisotropy has appeared regularly both in physics and in computer graphics [TAKA83] [YOKO88] [WARD92] : it consists to express the degree of anisotropy by an ellipsis of varying excentricity. Ward proposed such an anisotropic reflectance model based on two assumptions : a gaussian model is used for the specular part of the BDRF and an elliptical model is used for the anisotropic part [WARD92] :

$$\begin{cases} R_\lambda(t,v,v',w) = \dfrac{d}{\pi}\, D_\lambda + \dfrac{s}{4\pi\sqrt{vv'}}\, S_\lambda\ D(t,w) \quad \text{with} \\ D(t,w) = \dfrac{1}{mn}\ e^{\frac{t^2-1}{t^2}\left(\frac{w^2}{m^2}+\frac{1-w^2}{n^2}\right)} \quad \text{and} \quad d+s=1 \end{cases} \tag{8}$$

where

- $d \in [0,1]$, $s \in [0,1]$, $D_\lambda \in [0,1]$ and $S_\lambda \in [0,1]$ have the same meaning as in Equation 4
- $D(t,w) \in [0,\infty[$ has the same meaning as in Equation 5
- $m \in [0,0.5]$ (resp. $n \in [0,0.5]$) is the RMS slope of the surface in the x (resp. y) direction

4 Discussion

By examinating existing reflectance models, one can find several points that appear somewhat unsatisfactory. For instance, the BRDF is formulated as a linear combination with constant weights between a diffuse part and a specular one. The justification usually given by the authors is that, for a large class of materials, diffuse and specular components come from different physical phenomena, and thus they may have different colors. One classical example is a plastic surface (see Figure 2) on which light can be reflected either by the uncolored substrat in a coherent way (*ie* surface reflection is specular) or by the colored pigments beneath the surface in an incoherent way (*ie* subsurface reflection is diffuse) [COOK81].

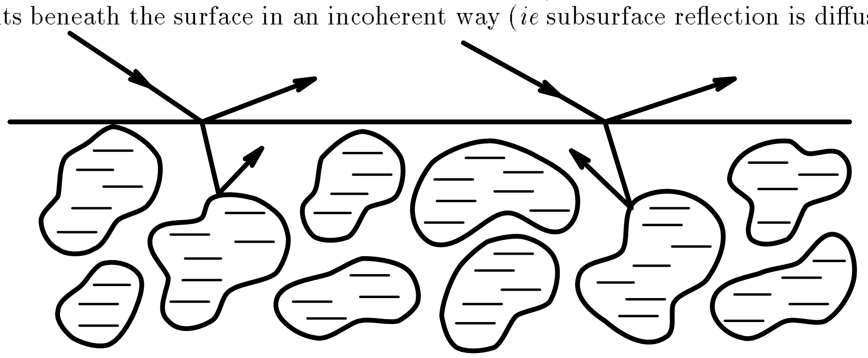


Figure 2 : Surface and subsurface reflection on a plastic material

But, as noticed by Shirley, such a linear combination with constant weights is incorrect because proportions of diffuse and specular components are usually not constant but function of the incident angle [SHIR90]. Taking the example of a varnished wood floor (see Figure 3), one can see that according to the Fresnel law, for large incident angles most light is reflected specularly by the varnish, whereas for small incident angles, most light penetrates the varnish before beeing reflected diffusely by the wood.

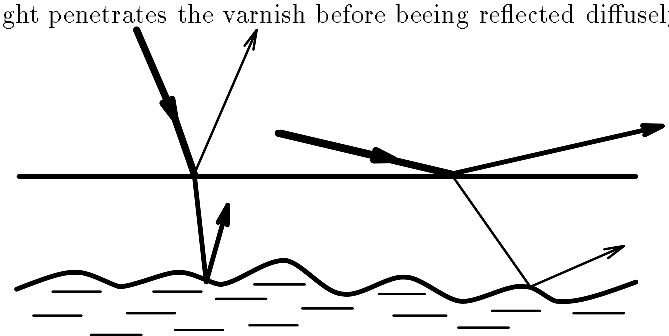


Figure 3 : Influence of the incident angle on surface and subsurface reflection

Beside these *heterogeneous* materials, there are a lot of *homogeneous* ones, for which the diffuse/specular distinction is unnecessary. For such materials (metals, for instance) there rather exists a kind of continuum between perfect diffuse and perfect specular behaviours (see Figure 4) according to the roughness of the surface. Therefore a linear combination with constant weights is inadequate again.

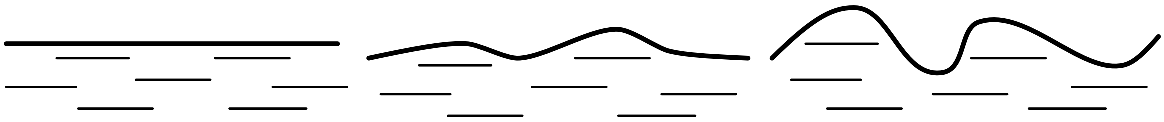


Figure 4 : Continuum between diffuse and specular for surface reflection

Another unappealing point in the existing models appears when light reaches or leaves a rough surface where self-obstruction can occur (see Figure 5). Usually, a geometrical attenuation coefficient (G in Equation 5) is used as a multiplicative factor to express the ratio of light that is not subject to that obstruction. But the remainder of the light (*ie* $1-G$) is reflected in other directions and not simply blocked. Currently, none of the existing reflectance models does correctly account for that reemission of self-obstructed light.

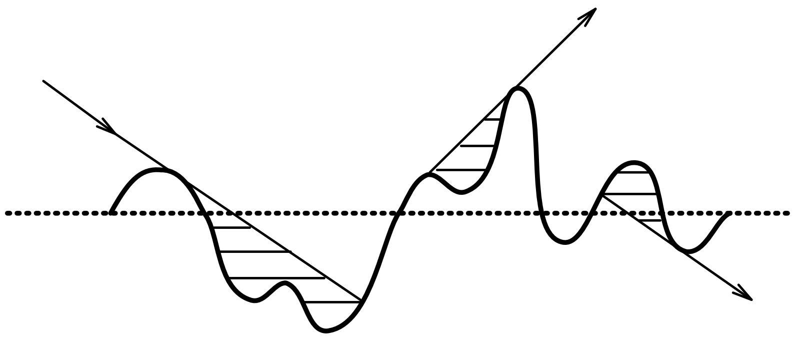


Figure 5 : No reemision for self-obstructed light

The last unsatisfactory point is about the accuracy/cost ratio. Empirical models [PHON75] [BLIN77] are inexpensive but lack of physical validity. For instance, they do not fulfill the normalization condition (Equation 3) and therefore the reflected energy is sometimes greater than the incident one.

On the other hand, complete theoretical models [KAJI85] [HE91] are physically accurate but imply an extremely high computational cost. Moreover, when including such a reflectance model in an image synthesis software, the error generated by other stages of the rendering pipeline (modeling, sampling, global illumination, interpolation) does usually totally cancel the benefit of greater accuracy : there is no need to compute BDRFs at a precision of 0.1%, if global illumination is only done at 5% and spectral sampling at 15%.

Ward has noticed such a contradiction [WARD92] and his model can be viewed as a kind of intermediary model, not searching for theoretical justification — except the Helmholtz Reciprocity Rule (Equation 2) and the Energy Conservation Law (Equation 3) — but for experimental justification. Therefore, the four parameters (d, s, m and n) of Equation 8 are not defined by hand but by least squares error minimization techniques, in order to fit experimental results as close as possible.

The model presented in the next section is also an intermediary model between empiricism and theory. With regards to the previous remarks, it is based on the following ideas :

- Main results of physics should be fulfilled *(Energy conservation law, Helmholtz reciprocity rule, Microfacet theory, Fresnel equation)*
- Continuum between lambertian and smooth surfaces should be created
- Distinction between homogeneous and heterogeneous materials should be made
- Isotropic and anisotropic behaviours should be taken in account

- Parameters should be specified either intuitively or related to experimental measurements
- Several levels of physical accuracy and computational cost should be provided

5 A Customizable Model

5.1 Parameters

In the new BDRF model, two different kinds of material are distinguished :

- SINGLE : Materials having homogeneous optical properties (metal, glass, paper, tissue)
- DOUBLE : Materials having heterogeneous optical properties (plastic, stratified or varnished surfaces) usually composed of a transparent layer over an opaque one, each of them beeing SINGLE materials.

The following parameters are used to caracterize a SINGLE material :

- $C_\lambda \in [0,1]$: Reflection factor for wavelength λ
- $r \in [0,1]$: Roughness factor ($r = 0$: perfect specular, $r = 1$: perfect diffuse)
- $p \in [0,1]$: Isotropy factor ($p = 0$: perfect anisotropy, $p = 1$: perfect isotropy)

For a DOUBLE material, a set of parameters is given for each layer, (C_λ, r, p) and (C'_λ, r', p').

The choice of these parameters was motivated mainly by two of there caracteristics. First, they can be understood intuitively and therefore easily defined by a non-physician user. Second, they can also be assigned by experimental measurements [WYSZ67] [PALI85]. Indeed, C_λ can be viewed as the reflectivity at normal incidence, r is related to the RMS slope of the surface, and p is the ratio of the RMS slopes between the scratch ($\varphi = 0$) and the ortho-scratch ($\varphi = \pi/2$) direction for an anisotropic surface.

5.2 Definition

Using notations of Section 2, we propose the following formulation for the new model :

$$\begin{cases} \text{SINGLE} & : R_\lambda(t,u,v,v',w) = S_\lambda(u)\ D(t,v,v',w) \\ \text{DOUBLE} & : R_\lambda(t,u,v,v',w) = S_\lambda(u)\ D(t,v,v',w) + [1 - S_\lambda(u)]\ S'_\lambda(u)\ D'(t,v,v',w) \end{cases} \tag{9}$$

where $S_\lambda(u)$ (resp $D(t,v,v',w)$) expresses the spectral (resp directional) behaviour of the BRDF. Several formulations (more or less expensive and more or less accurate) for the two factors are given below.

5.3 Spectral factor

The simplest expression for the spectral factor is to consider it as a constant function :

$$S_\lambda(u) = C_\lambda \tag{10}$$

But in fact, $S_\lambda(u)$ is function of the incident angle and should obey to Fresnel law. Rather than using the true formulation of the Fresnel factor, we propose Equation 11 which is a close approximation, as shown in [SCHL92] :

$$S_\lambda(u) = C_\lambda + (1 - C_\lambda)\ (1-u)^5 \tag{11}$$

5.4 Directional factor

A first formulation for the directional factor arises from a straigthforward extension of [BECK63] :

$$D(t,v,v',w) = \frac{1}{4\pi v v'}\ Z(t)\ A(w) \tag{12}$$

where $Z(t)$ (resp $A(w)$) expresses the zenith angle α (resp azimuth angle φ) dependence.

For physical validity, $D(t, v, v', w)$ has to satisfy Equation 2 and Equation 3. Obviously, D fulfills the reciprocity condition. On the other hand, one can show [SCHL92] that D fulfills the normalization condition when :

$$\int_0^1 2t\ Z(t)\ dt = 1 \qquad \text{and} \qquad \int_0^1 \sqrt{1-w^2}\ A(w)\ dw = \frac{\pi}{2} \tag{13}$$

Z and A can be mixed in a gaussian elliptical function as in [WARD92]. We propose a separable expression that also satisfies Equation 13 :

$$Z(t) = \frac{r}{(1 + rt^2 - t^2)^2} \qquad \text{and} \qquad A(w) = \sqrt{\frac{p}{p^2 - p^2 w^2 + w^2}} \tag{14}$$

Figure 6a (resp 6b) shows $Z(\alpha)$ (resp $A(\varphi)$) in polar coordinates for various values of r (resp p). Notice that when $r = 1$, $Z(t)$ is a constant function (perfect diffuse) and when $r = 0$, $Z(t)$ becomes a Dirac function (perfect specular). The same remark can be made for $A(w)$ which varies continuously between a constant function when $p = 1$ (perfect isotropy) and a Dirac function when $p = 0$ (perfect anisotropy).

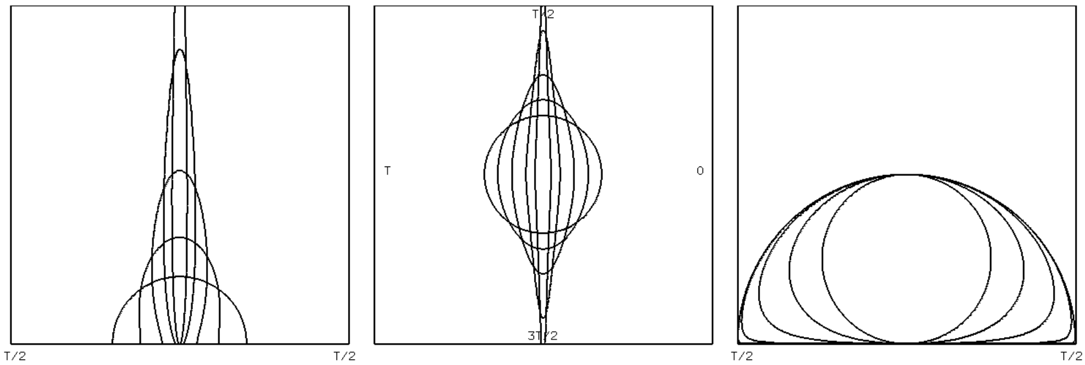


Figure 6 : Directional factor in logarithmic polar coordinates
(a) Zenith angle dependence $Z(t)$ for $r = 0.01, 0.05, 0.2, 0.5, 1.0$
(b) Azimuth angle dependence $A(w)$ for $p = 0.01, 0.05, 0.2, 0.5, 1.0$
(c) Geometrical obstruction factor $G(v)$ for $r = 0.01, 0.05, 0.2, 0.5, 1.0$

As in [SMIT67], self-shadowing without reemission can be included by a geometrical obstruction factor $G(v)G(v')$ where $G(v)$ (resp $G(v')$) expresses the ratio of reflected (resp incident) non obstructed light :

$$D(t, v, v', w) = \frac{G(v)G(v')}{4\pi v v'}\ Z(t)\ A(w) \tag{15}$$

Rather than using the true formulation of the Smith factor, we propose Equation 16 which is a close approximation, as shown in [SCHL92] (see Figure 6c) :

$$G(v) = \frac{v}{r - rv + v} \qquad \text{and} \qquad G(v') = \frac{v'}{r - rv' + v'} \tag{16}$$

Due to the presence of v and v' on the denominator, Equation 12 does not provide complete transition from perfect specular to perfect diffuse. This restriction can be removed by providing a linear interpolation, according to the roughness factor, between the BRDF of Equation 12 and a lambertian BRDF :

$$D(t, v, v', w) = \frac{r}{\pi}\ A(w)\ +\ \frac{1-r}{4\pi v v'}\ Z(t)\ A(w) \tag{17}$$

And finally, self-shadowing with reemission can be included by relating the two weights of Equation 17 to the geometrical obstruction factor, as explained in section 4 :

$$D(t, v, v', w) = \frac{1 - G(v)G(v')}{\pi}\ A(w)\ +\ \frac{G(v)G(v')}{4\pi v v'}\ Z(t)\ A(w) \tag{18}$$

6 Results

In order to show varying illumination effects (incidence angles ranging from grazing to normal and varying either fast or slow...) a simple test scene composed of cylinders has been chosen, inspired from [HE91]. To achieve a better understanding of the behaviour of the new model, only direct illumination from a single light source put at the view point is shown. Every cylinder on Figure 7 and 8 is made of an homogeneous material and has been rendered individually at a 256x512 resolution using stochastic ray-tracing.

Figure 7 illustrates the continuum that is achieved between diffuse and specular reflection by taking four different values for r. Figure 8 shows the continuum that is provided between isotropy and anisotropy by taking four different value for p. In order to exhibit anisotropy, the cylinder has been made of brushed metal, having concentrical circular scratches on its top and parallel horizontal scratches on its face.

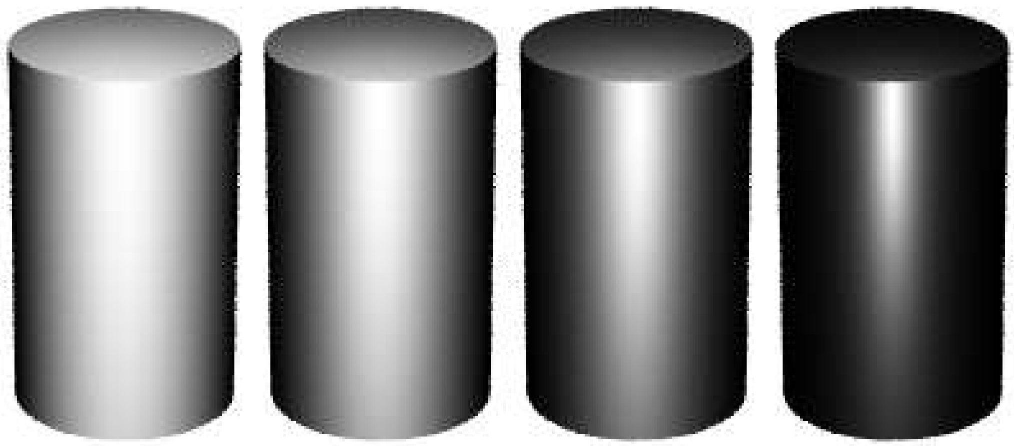


Figure 7 : Continuum between diffuse and specular reflection
(a) $r = 1.0$ $p = 1.0$ (b) $r = 0.5$ $p = 1.0$ (c) $r = 0.2$ $p = 1.0$ (d) $r = 0.05$ $p = 1.0$

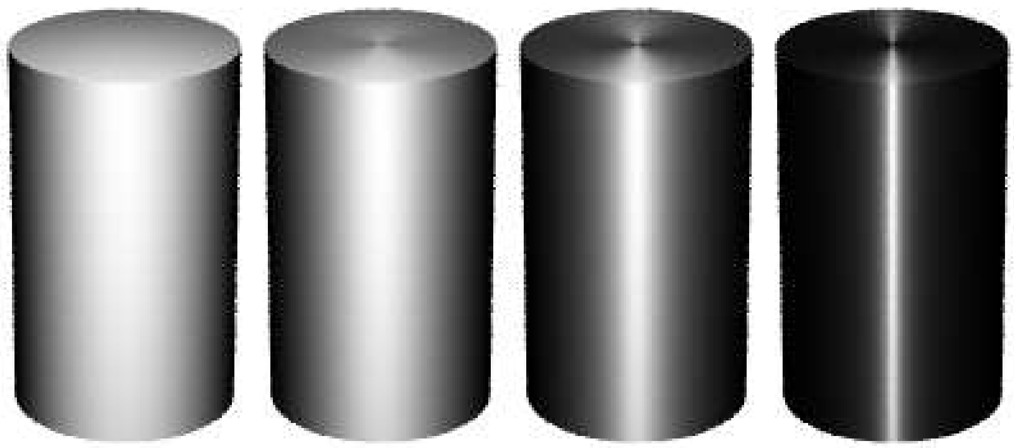


Figure 8 : Continuum between isotropic and anisotropic reflection
(a) $r = 1.0$ $p = 1.0$ (b) $r = 1.0$ $p = 0.5$ (c) $r = 1.0$ $p = 0.2$ (d) $r = 1.0$ $p = 0.05$

7 Conclusion

A BRDF model for computer graphics including the following features has been presented :

- A distinction is made between materials with homogeneous properties and materials with heterogeneous properties (which are supposed to be composed of two homogeneous layers).

- A two dimensional continuum is insured both between perfect diffuse and perfect specular, and between perfect isotropy and perfect anisotropy.
- The Fresnel factor is introduced in a new way, expressing the weights of surface vs subsurface reflection.
- The obstruction factor is also introduced in a new way, expressing the weights of single vs multiple reflection.
- Very few parameters are used to define a material, each of them beeing intuitive and related to experimental data usually available.
- A formulation of varying complexity is provided, allowing the user to taylor physical accuracy vs computational cost.
- Every proposed formulation obeys to reciprocity and energy conservation laws and therefore physically inconsistant results are avoided.
- Finally, every proposed formulation uses very simple expressions (only basic arithmetic operators $(+ - * /)$, except a square root for anisotropy) and therefore eventual hardware implementations are enabled.

Aknowledgment

Special thanks to G.Ward, F.Sillion and K.Bouatouch for their numerous comments during the development of the model specifications.

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